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Overall spatial characterization of nonparaxial radially polarized beams propagating from the focal plane of a high-focusing optical system

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Abstract
The spatial structure of nonparaxial radially polarized fields, propagating from the focal plane of a high-focusing optical system, is analytically characterized by means of certain overall parameters. The procedure is based in a formal analogy with the irradiance-moments framework used for describing paraxial beams. The dependence on the propagation distance from the focal plane is derived for the effective beam width recently defined for this type of vectorial highly focused field. Due to the different behavior of the transverse and longitudinal components of the field, they are investigated in separate ways. We also introduce for this kind of nonparaxial field the concepts of beam waist (plane where the transverse beam size reaches the minimum value) and collimation coefficient (connected with the beam spreading). The suitability of the proposed global description is examined by considering, at each transverse plane, the power-content ratio within a circle whose radius is the beam width. The above results are applied to an illustrative example.

Keywords: nonparaxial fields, radially polarized fields, spatial beam characterization, highly-focused beams

1. Introduction

Highly focused light beams are currently used in a large number of applications in nanooptics [1]. More specifically, radially polarized beams (see, for example [2–6], and references therein) have been revealed to be particularly useful because, at the focal region, their spot sizes are smaller than the widths shown by conventional linearly polarized fields [3–5]. In these experiments, the beam width is associated with a local value of the irradiance, namely, the distance between opposite points whose irradiance is half the maximum value of the irradiance profile. Another standard way of describing the transverse beam size involves the contour line of constant irradiance containing half the power.

The above definitions allow a value for the beam size to be assigned from the experimental spatial profile. In the present work, however, attention will be centered on the development of a general analytical spatial description with the following goals.

(i) A beam width definition should be valid for arbitrary amplitude distributions. In addition, it should be capable of comparing the overall characteristics of beams with arbitrary profiles. Note, for instance, that, when the transverse profile exhibits ripples, the beam size based on half the central peak value of the irradiance could not be representative enough, because the power content inside such a region is not considered.

(ii) It would also be useful that changes of the beam width upon free propagation obey a simple analytical law. As we will show later, an incident field whose amplitude takes complex values at the input plane of the focusing device does not reach the waist (minimum beam width) at the focal plane $z = 0$ of such an optical system. This would give rise to a shift of the peak-power plane, which could analytically be determined if we know the z-dependence of the beam width.

(iii) The changes suffered by the beam width upon propagation of the field would allow the description of the beam spreading. This possibility increases the interest of establishing a beam-width definition whose evolution with z could be analytically determined. Concerning the two definitions mentioned at the beginning of this section, note that the value of such beam widths at a certain transverse plane cannot be analytically inferred from their value at another plane.

As is well known, the behavior summarized in (i)–(iii) is fulfilled by the ISO standard definition of beam width in the paraxial regime. This definition involves the overall spatial shape over the beam cross-section, and is closely connected with the power-content ratio inside the spot radius. The paraxial beam width is based on the so-called irradiance-moments formalism [7–17], which has been adopted to define a number of ISO standards for laser beams [18]. In addition, it has been shown that the paraxial width is modified under field propagation into free space according to a simple quadratic law in terms of the distance z from the focal plane of the focusing system.

Unfortunately, when one tries to extend this paraxial definition to highly focused (nonparaxial) fields, certain integrals do not converge and the definition fails. To our knowledge, only recently [19] has an analytical definition been proposed to represent the transverse size at the focal plane of a high-focusing system with an incident radially polarized field. This definition was established on the basis of a formal analogy with the irradiance-moments framework. Due to the inherent vectorial character of light, the transverse and longitudinal field components were considered separately. Although preliminary results [19] confirm the suitability of the proposed definition, several significant issues related to this problem remain to be addressed, namely,

- the beam-width definition should be extended to any transverse plane after the focal plane of the optical system,
- the validity of a quadratic law for the z-dependence of the beam width should be investigated,
- the position of the waist plane should be analytically determined from the value of the field amplitude at the initial plane,
- the beam spreading upon propagation of this kind of highly focused fields should be studied,
- it should be checked that the power-content ratio within a circular region around the propagation axis, whose radius is given in terms of the beam width, reaches high enough values at any transverse plane (not only at the focal plane of the system).

The aim of the present work is to investigate the above issues. Thus, the paper is organized as follows. In section 2, the beam-width definition is provided at any transverse plane. Longitudinal and transverse components are considered, along with the whole field. The evolution of the beam width with the distance of propagation is also derived, and the concepts of beam waist and collimation coefficient are established. In all the cases, the expressions are given in terms of both the filling factor and the angular aperture of the high-focusing optical device. An illustrative example is studied in section 3, which includes and discusses the evolution of the power-content ratio upon propagation. Finally, the main conclusions are summarized in section 4.

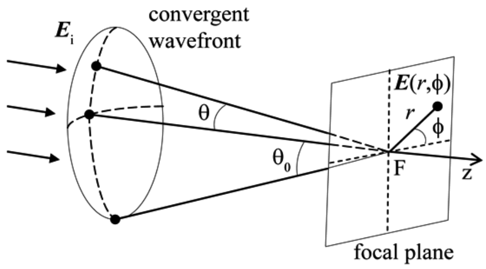

Figure 1. An illustration of the notation and the geometry of the problem. The value θ_0 corresponds to the semi-aperture angle of the aplanatic system.

2. Key definitions and propagation from the focal plane

Let us consider a monochromatic radially polarized field at the input plane of an aplanatic high-focusing optical system. The electric field vector $\boldsymbol{E}$ at a transverse plane z of the optical system ($z = 0$ defines the focal plane) can be obtained from the following expression (see, for instance, [1, 20]):

$$\boldsymbol{E}(r,\phi,z) = A\int_0^{\theta_0}\int_0^{2\pi} E_i(\theta)\sqrt{\cos\theta}\begin{pmatrix}\cos\theta\cos\varphi\\ \cos\theta\sin\varphi\\ -\sin\theta\end{pmatrix} \times \exp[ikr\sin\theta\cos(\phi-\varphi)]\exp(ikz\cos\theta)\sin\theta\,\mathrm{d}\theta\,\mathrm{d}\varphi, \quad (1)$$

where A is a constant, proportional to $\frac{\mathrm{i}}{\lambda}$ (see [1]), whose explicit value is not required in the calculations, E_i represents the field amplitude (incident on the system) assumed to be, for simplicity, rotationally symmetric around the propagation axis z; r and ϕ denote the polar coordinates at the transverse plane, and the angles θ and θ_0 are shown in figure 1. In equation (1) the vector inside the integral provides the vectorial structure of an incident pure radially polarized field, and the factor $\sqrt{\cos\theta}$ follows from the intensity law that ensures that the energy incident on an aplanatic optical system equals the energy that leaves the system. Note also that the factor $\exp(ikz\cos\theta)$ in equation (1) enables us to determine the propagated field at a distance z from the focal plane.

After writing, for convenience, $\rho \equiv \sin\theta$, in the rest of the paper the incident field amplitude E_i will be written in the form

$$E_i(\rho) = \rho h(\rho), \quad (2)$$

where $h(\rho)$ denotes an arbitrary function independent of φ.

2.1. Characterization of the transverse component

Let us first study the transverse part $\boldsymbol{E}_\mathrm{T}$ of the field at the focal plane. It has been shown [19] that $\boldsymbol{E}_\mathrm{T}(z=0)$ can be written in the form

$$\boldsymbol{E}_\mathrm{T}(z=0) \equiv \begin{pmatrix} E_x(r,\phi) \\ E_y(r,\phi) \end{pmatrix} = 2\pi \mathrm{i} A \begin{pmatrix} \cos\phi \\ \sin\phi \end{pmatrix} f(r), \quad (3)$$

with

$$f(r) = \frac{a^2}{kr} F(a) J_2(kra) - \frac{1}{kr}\int_0^a \rho^2 J_2(kr\rho)\frac{\mathrm{d}F(\rho)}{\mathrm{d}\rho}\,\mathrm{d}\rho, \quad (4)$$

where $a \equiv \sin\theta_0$ takes into account the angular aperture of the focusing system, and

$$F(\rho) = h(\rho)(1-\rho^2)^{1/4}. \quad (5)$$

Note that, in equation (3), $\left(\begin{smallmatrix}\cos\phi\\ \sin\phi\end{smallmatrix}\right)$ is a radial vector. Consequently, the transverse field $\boldsymbol{E}_\mathrm{T}$ exhibits a radial polarization.

On the basis of a formal analogy with the irradiance-moments formalism, a definition for the beam width at the focal plane of the focusing system was recently proposed for this kind of highly focused fields [19]. In fact, the (squared) width w_T^2 of the transverse field $\boldsymbol{E}_\mathrm{T}$ at the focal plane $z=0$ was defined in the form

$$w_\mathrm{T}^2 = \frac{1}{k^2 I_{0T}}\left[\frac{a^2c_2^2}{4}|F(a)|^2 + \int_0^a \rho^2\left|\frac{\mathrm{d}F(\rho)}{\mathrm{d}\rho}\right|^2\rho\,\mathrm{d}\rho\right], \quad (6)$$

where

$$I_{0T} = \int_0^a |\rho h(\rho)(1-\rho^2)^{1/4}|^2\rho\,\mathrm{d}\rho, \quad (7)$$

and c_2 denotes the second zero of the Bessel function J_2. To derive the generalized beam-width definition at any other plane z, we will make use of the propagation factor $\exp(\mathrm{i}kz\cos\theta)$ inside equation (1). Thus, function $F(\rho)$ becomes at plane z

$$\begin{aligned} F(\rho,z) &= h(\rho)(1-\rho^2)^{1/4}\exp(\mathrm{i}kz\cos\theta) \\ &= h(\rho)(1-\rho^2)^{1/4}\exp\left(\mathrm{i}kz\sqrt{1-\rho^2}\right), \end{aligned} \quad (8)$$

and the width $w_\mathrm{T}^2(z)$ of the transverse field component reads

$$w_\mathrm{T}^2(z) = \frac{1}{k^2 I_{0T}}\left[\frac{a^2c_2^2}{4}|F(a,z)|^2 + \int_0^a \rho^2\left|\frac{\partial F(\rho,z)}{\partial\rho}\right|^2\rho\,\mathrm{d}\rho\right]. \quad (9)$$

It is important to remark that $w_\mathrm{T}^2(z)$ reduces to the conventional beam-width definition for paraxial fields when $F(a,z)=0$. On the other hand, note that the first term of equation (9) does not depend on ρ. In addition, we have

$$\frac{\partial F(\rho,z)}{\partial\rho} = \left[\frac{\partial F(\rho,0)}{\partial\rho} - \frac{\mathrm{i}k\rho z}{\sqrt{1-\rho^2}}F(\rho,0)\right] \times \exp\left(\mathrm{i}kz\sqrt{1-\rho^2}\right), \quad (10)$$

so that

$$\left|\frac{\partial F(\rho,z)}{\partial\rho}\right|^2 = \left|\frac{\partial F(\rho,0)}{\partial\rho}\right|^2 + \frac{k^2\rho^2z^2}{1-\rho^2}|F(\rho,0)|^2 + \frac{2k\rho z}{\sqrt{1-\rho^2}}\,\mathrm{Im}\left[F(\rho,0)\frac{\partial F^*(\rho,0)}{\partial\rho}\right]. \quad (11)$$

After substituting this expression in equation (9), we obtain for the width of the transverse component $\boldsymbol{E}_\mathrm{T}$ a simple quadratic law with the distance z:

$$w_\mathrm{T}^2(z) = w_\mathrm{T}^2(0) + \Phi_\mathrm{T}^2 z^2 + 2\Psi_\mathrm{T} z, \quad (12)$$

where

$$\begin{aligned} \Phi_\mathrm{T}^2 &= \frac{1}{I_{0T}}\int_0^a \frac{\rho^4}{1-\rho^2}|F(\rho,0)|^2\rho\,\mathrm{d}\rho \\ &= \frac{1}{I_{0T}}\int_0^a \frac{\rho^4}{\sqrt{1-\rho^2}}|h(\rho)|^2\rho\,\mathrm{d}\rho, \end{aligned} \quad (13a)$$

$$\begin{aligned} \Psi_\mathrm{T} &= \frac{1}{kI_{0T}}\int_0^a \frac{\rho^3}{\sqrt{1-\rho^2}}\,\mathrm{Im}\left[F(\rho,0)\frac{\partial F^*(\rho,0)}{\partial\rho}\right]\rho\,\mathrm{d}\rho \\ &= \frac{1}{kI_{0T}}\int_0^a \rho^3\,\mathrm{Im}\left[h(\rho)\frac{\mathrm{d}h^*(\rho)}{\mathrm{d}\rho}\right]\rho\,\mathrm{d}\rho. \end{aligned} \quad (13b)$$

A number of general conclusions follow from equations (12) to (13*b*).

(a) Equation (12) is a quadratic function of the propagation distance z, as occurs in the conventional paraxial case [8–13].

(b) The squared width $w_\mathrm{T}^2(z)$ reaches the minimum value (waist plane) at a distance z_Tw from the focal plane, where

$$z_\mathrm{Tw} \equiv -\frac{\Psi_\mathrm{T}}{\Phi_\mathrm{T}^2}. \quad (14)$$

(c) In general, the waist plane is not the focal plane, and the shift is given by equation (14). However, in the literature (see, for example [1]), it is assumed that the waist of the incident field coincides with the lens of the focusing optical system, and, consequently, the beam hits such a lens with a planar phase front, so the incoming field takes real values. In such a case, the waist plane and the focal plane are identical, and no shift would appear.

It is apparent from equation (12) that the beam spreads after reaching its waist. In order to quantitatively estimate how fast (in distance z) the field expands from the focal plane we can evaluate the function $\frac{\mathrm{d}^2(w_\mathrm{T}^2)}{\mathrm{d}z^2}$ at plane $z=0$. We get at once

$$\frac{\mathrm{d}^2(w_\mathrm{T}^2)}{\mathrm{d}z^2} = 2\Phi_\mathrm{T}^2. \quad (15)$$

This value would then be a measure of the stability of the beam width around the waist plane, and we refer to it as the collimation coefficient. It is important to remark that, although equation (12) resembles the quadratic paraxial propagation law of the (squared) width (calculated from the irradiance moments), special care should, however, be taken in order to avoid any direct identification of the collimation coefficient with the far-field divergence of a paraxial beam.

2.2. Characterization of the longitudinal component

Let us now consider the longitudinal component E_z, whose (squared) beam width at plane $z=0$ was recently defined in the form [19]

$$w_\mathrm{L}^2 = \frac{1}{k^2 I_{0L}}\left[\frac{c_1^2}{2}|G(a)|^2 + \int_0^a \left|\frac{\partial G(\rho)}{\partial\rho}\right|^2\rho\,\mathrm{d}\rho\right], \quad (16)$$

where c_1 denotes the first zero of the Bessel function J_1, and

$$G(\rho) = \frac{\rho^2 h(\rho)}{(1-\rho^2)^{1/4}}, \tag{17a}$$

$$I_{0\mathrm{L}} = \int_0^a \left| \frac{\rho^2 h(\rho)}{(1-\rho^2)^{1/4}} \right|^2 \rho \, \mathrm{d}\rho. \tag{17b}$$

From equation (1), it then follows for $E_z(z)$

$$E_z(r,\phi,z) = -2\pi A \int_0^a G(\rho,z) J_0(kr\rho) \times \exp\left[\mathrm{i}kz\sqrt{1-\rho^2} \right] \rho \, \mathrm{d}\rho, \tag{18a}$$

with

$$G(\rho,z) = \frac{\rho^2 h(\rho)}{(1-\rho^2)^{1/4}} \exp\left(\mathrm{i}kz\sqrt{1-\rho^2} \right). \tag{18b}$$

Accordingly, in a similar way to that followed for the transverse field component, we can generalize the (squared) beam width of the longitudinal component at any plane, namely,

$$w_\mathrm{L}^2(z) = \frac{1}{k^2 I_{0\mathrm{L}}} \left[\frac{c_1^2}{2} |G(a,z)|^2 + \int_0^a \left| \frac{\partial G(\rho,z)}{\partial \rho} \right|^2 \rho \, \mathrm{d}\rho \right], \tag{19}$$

which can be written in the form

$$w_\mathrm{L}^2(z) = w_\mathrm{L}^2(0) + \Phi_\mathrm{L}^2 z^2 + 2\Psi_\mathrm{L} z, \tag{20}$$

with

$$\Phi_\mathrm{L}^2 = \frac{1}{I_{0\mathrm{L}}} \int_0^a \frac{\rho^6 |h(\rho)|^2}{(1-\rho^2)^{3/2}} \rho \, \mathrm{d}\rho, \tag{21a}$$

$$\Psi_\mathrm{L} = \frac{1}{k I_{0\mathrm{L}}} \int_0^a \frac{\rho^5}{1-\rho^2} \mathrm{Im}\left[h(\rho) \frac{\mathrm{d}h^*(\rho)}{\mathrm{d}\rho} \right] \rho \, \mathrm{d}\rho. \tag{21b}$$

Equations (20)–(21*b*) are the counterparts of equations (12)–(13*b*), now concerning the longitudinal component, and therefore conclusions (a)–(c) also apply here. More specifically, the transverse plane z_{Lw} where the longitudinal component reaches a minimum is now given by the expression

$$z_{\mathrm{Lw}} = -\frac{\Psi_\mathrm{L}}{\Phi_\mathrm{L}^2}. \tag{22}$$

It is clear from this equation that, in general, $z_{\mathrm{Tw}} \neq z_{\mathrm{Lw}}$. However, it should be remarked that both waists are placed at the focal plane if the incident light field is real at the input plane of the focusing system.

As for the transverse component, a collimation coefficient can also be defined in the form

$$\frac{\mathrm{d}^2(w_\mathrm{L}^2)}{\mathrm{d}z^2} = 2\Phi_\mathrm{L}^2, \tag{23}$$

with the same remark as pointed out after equation (15) for the transverse case. Also note that, in general, the field components spread in a different way.

2.3. Characterization of the global field

Let us finally introduce the width $w_\mathrm{G}^2(z)$ of the global field. By taking into account that

$$w_\mathrm{G}^2 = \frac{I_\mathrm{T} w_\mathrm{T}^2 + I_\mathrm{L} w_\mathrm{L}^2}{I}, \tag{24}$$

where

$$I_\mathrm{T}(z) \equiv \int_0^\infty \int_0^{2\pi} |\boldsymbol{E}_\mathrm{T}(r,\phi,z)|^2 r \, \mathrm{d}r \, \mathrm{d}\phi, \tag{25a}$$

$$I_\mathrm{L}(z) \equiv \int_0^\infty \int_0^{2\pi} |E_z(r,\phi,z)|^2 r \, \mathrm{d}r \, \mathrm{d}\phi, \tag{25b}$$

with $I_\mathrm{G} = I_\mathrm{T} + I_\mathrm{L}$, we have

$$w_\mathrm{G}^2(z) = \frac{1}{k^2 I_{0\mathrm{G}}} \left\{ \frac{a^2 c_2^2}{4} |F(a,z)|^2 + \frac{c_1^2}{2} |G(a,z)|^2 + \int_0^a \left[\rho^2 \left| \frac{\partial F(\rho,z)}{\partial \rho} \right|^2 + \left| \frac{\partial G(\rho,z)}{\partial \rho} \right|^2 \right] \rho \, \mathrm{d}\rho \right\}. \tag{26}$$

As for z_{Tw} and z_{Lw}, the value of the width of the whole field at any plane z also satisfies a quadratic law, i.e.

$$w_\mathrm{G}^2(z) = w_\mathrm{G}^2(0) + \Phi_\mathrm{G}^2 z^2 + 2\Psi_\mathrm{G} z, \tag{27}$$

where now

$$\Phi_\mathrm{G}^2 = \frac{1}{I_0} \int_0^a \frac{\rho^4 |h(\rho)|^2}{(1-\rho^2)^{3/2}} \rho \, \mathrm{d}\rho, \tag{28a}$$

$$\Psi_\mathrm{G} = \frac{1}{k I_0} \int_0^a \frac{\rho^3}{1-\rho^2} \mathrm{Im}\left[h(\rho) \frac{\mathrm{d}h^*(\rho)}{\mathrm{d}\rho} \right] \rho \, \mathrm{d}\rho, \tag{28b}$$

and $I_0 \equiv I_{0\mathrm{T}} + I_{0\mathrm{L}}$. The waist plane of the whole beam is placed at plane z_{Gw} given by

$$z_{Gw} = -\frac{\Psi_\mathrm{G}}{\Phi_\mathrm{G}^2}, \tag{29}$$

which, in general, differs from z_{Tw} and z_{Lw}. Finally, the collimation coefficient now reads

$$\frac{\mathrm{d}^2(w_\mathrm{G}^2)}{\mathrm{d}z^2} = 2\Phi_\mathrm{G}^2. \tag{30}$$

3. Application to an example: evolution of the power-content ratio under propagation

Let us now apply the above analytical results to the particular but illustrative case of an incident field amplitude E_i given by

$$E_i(\rho) = \rho h(\rho) = \rho \exp\left(-\frac{f^2 \rho^2}{\omega_0^2} \right) = \rho \exp\left(-\frac{\rho^2}{f_0^2 a^2} \right), \tag{31}$$

where f_0 represents the so-called filling factor, which, for this field, can be defined in the usual way [1], namely, $f_0 = \omega_0 (f \sin\theta_0)^{-1}$, f being the focal length of the focusing system.

In order to test the validity of the quadratic law for the z-dependence of the beam width, we have to evaluate, at each transverse plane, the radius of the region (around the z-axis) where the power is concentrated. It was pointed out in [19] that, for paraxial beams, the radius of the region that assures 75% of the total power should be twice the root of the second-order irradiance moment (see also [21]). By adopting this formal analogy, high enough values of the power-content ratio

$$P_{2w_j}(z) = \frac{\int_0^{2w_j} I_j(r,z) r\,\mathrm{d}r}{\int_0^{\infty} I_j(r,z) r\,\mathrm{d}r}, \qquad j = \mathrm{T, L, G}, \qquad (32)$$

would confirm the overall description upon propagation. In equation (32) the subscripts T, L and G refer to the transverse, longitudinal and global field, respectively, and $I_j(r,z)$, j = T, L, G, denotes the (rotationally symmetric) irradiance distribution of the beam profile at each transverse plane z.

Figure 2 shows (a proportionality factor k apart) the evolution of the widths $2w_\mathrm{T}$, $2w_\mathrm{L}$ and $2w_\mathrm{G}$ upon propagation, for three values of the filling factor f_0.

The value $z = 0$ corresponds to the focal plane, and each curve has been plotted in the range of distances $[0, \tilde{z}_j]$, j = T, L, G, where $\tilde{z}_j$ is the distance that the field component should travel to increase its width by a factor $\sqrt{2}$ with regard to the value at the focal plane, i.e., $2w_j(z = \tilde{z}) = 2\sqrt{2}w_j(z = 0)$. This means that, at the plane $z = \tilde{z}$, the area of the circle with radius $w_j(\tilde{z})$ is twice the area of the circular region at $z = 0$ whose radius is $w_j(0)$. We see from the figures that, in all the cases, the beam expands from the focal plane (where the field amplitude is real). This constitutes a similar behavior to that exhibited by conventional beams. Moreover, it also follows from the figures that both the beam width and the collimation coefficient increase with f_0. In addition, the widths of the transverse and longitudinal field components have the same order of magnitude, whereas the ratio $\frac{\Phi_\mathrm{L}^2}{\Phi_\mathrm{T}^2}$ approaches 2. It should be remarked that the above properties apply for a wide range of values of the filling factor.

Figure 3 allows us to test whether the power is concentrated on a circle (around the z-axis) whose radius is $2w_j$, j = T, L, G. The curves plot the results for the values f_0 = 0.6, 1 and 1.4. In all the cases, we see that P_{2w} is always higher than 88% of the total power. Moreover, this ratio improves when the beam propagates from the focal plane.

4. Conclusions

The evolution of the transverse size (represented by the beam width) of nonparaxial radially polarized fields has been shown to obey a quadratic law in terms of the distance z from the focal plane of a high-focusing optical system. Use has been made of a formal analogy with the standard irradiance moments that describe the overall spatial structure of paraxial fields. In addition, it has been shown that the position of the beam waist (minimum width) can be obtained from the field structure at the input plane. It has been seen that, in general, the waist plane does not match the focal plane of the focusing optical system. This shift has been analytically determined. Furthermore, the beam spreading after the waist has been analytically characterized by the so-called collimation coefficient. It should be remarked that, although in the quadratic dependence with z of the beam width this parameter formally resembles the well-known far-field divergence of a paraxial field, in the present case we have to be cautious with such an analogy.

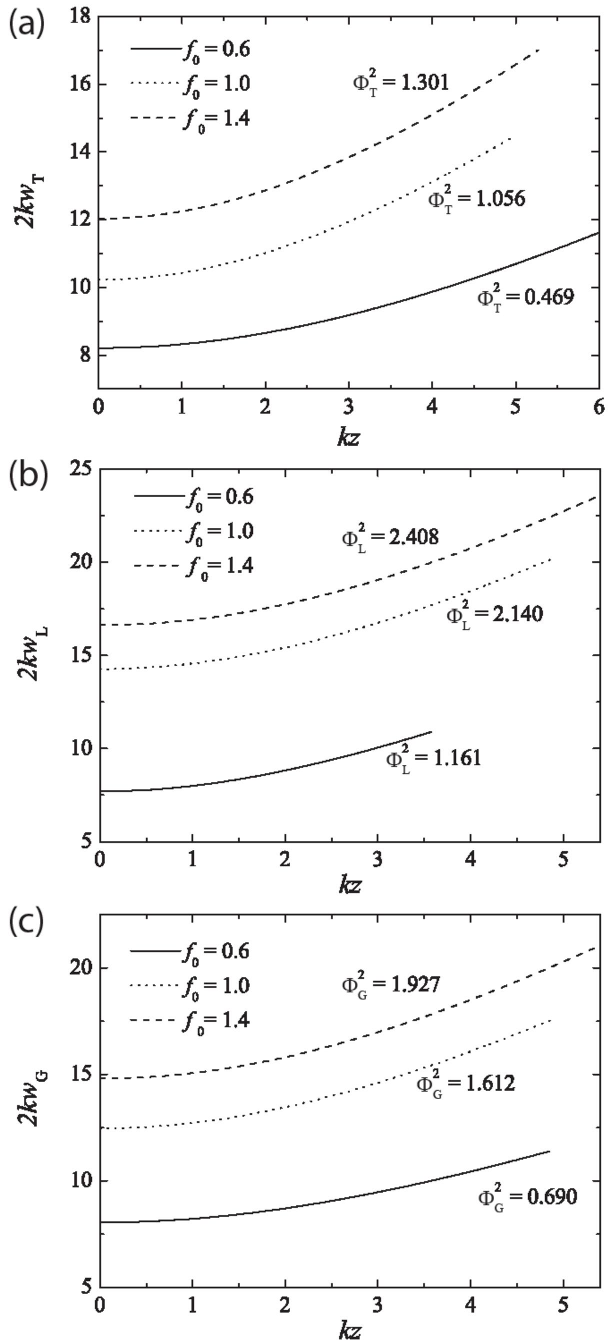

Figure 2. Evolution with z of the dimensionless parameter $2\,kw$, associated with the transverse component (a), longitudinal component (b), and whole field (c) for several values of the filling factor. Abscissas are written in terms of the (dimensionless) distance kz from the focal plane $z = 0$. The values of the collimation coefficient Φ^2 are given for each curve.

Due to the vectorial character of the light field, the transverse and longitudinal components of the field should be investigated in separate ways. Moreover, it has been shown that the beam size, the location of the waist plane, and the collimation coefficient are, in general, different for transverse and longitudinal field components.

At each transverse plane z, the power-content ratio within a circle whose radius is the proposed beam width has been examined by means of an illustrative example. High enough

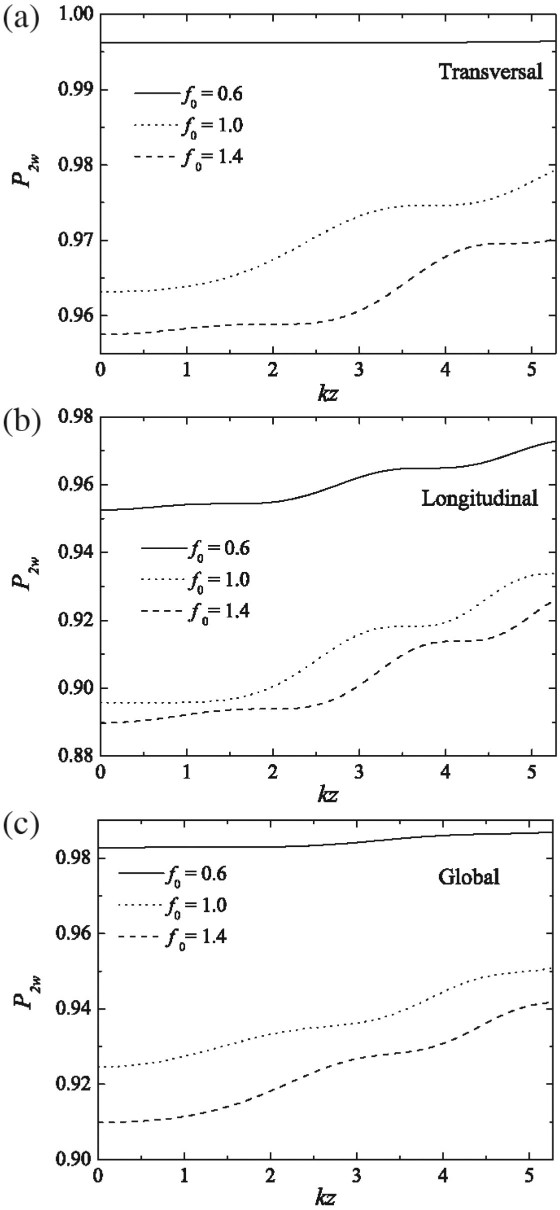

Figure 3. Power-content ratio P_{2w}, defined by equation (32), in terms of kz for several values of f_0 for the transverse component (a), the longitudinal component (b), and the whole field (c).

values of this ratio have been obtained upon propagation, a result that confirms the suitability of these definitions for characterizing and comparing the overall spatial structure of this kind of nonparaxial beam.

It should also be remarked that the procedure used in the present paper can be extended in a similar manner to deal with azimuthally polarized beams, which can be considered, in a sense, as the counterpart of radially polarized fields. In fact, for azimuthally polarized light, the vector inside the integral in equation (1) would read $(-\sin\varphi, \cos\varphi, 0)$, and, consequently, the longitudinal component E_z of azimuthally polarized fields will vanish at any plane. Moreover, for these fields, equation (3) becomes

$$E_T(z=0) \equiv 2\pi i A \begin{pmatrix} -\sin\phi \\ \cos\phi \end{pmatrix} f(r), \tag{33}$$

where $f(r)$ is given by equation (4), but now the function $F(\rho)$ should be written in the form (compare with equation (5))

$$F(\rho) = \frac{h(\rho)}{(1-\rho^2)^{1/4}}. \tag{34}$$

With such changes, the procedure would follow similar steps to that considered in section 2.

Let us finally point out the interest in generalizing the above parameters to almost 'pure' longitudinally polarized fields (based, for instance, on non-diffracting beams) that exhibit sub-diffraction beam size (see [22] and references herein). These fields are attracting increasing attention and have been used for particle acceleration and fluorescent microscopy, among other applications. Thus, this subject deserves study in the future.

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